Continuous Optimisation: Chpt 5 Exercises

October 31, 2016

1. For $\mathbf{c}, \mathbf{a}_1, \ldots, \mathbf{a}_m \in \mathbb{R}^n$, $\mathbf{b} \in \mathbb{R}^m$ consider the following two optimisation problems:

$$\min_{\mathbf{y} \in \mathbb{R}^m} \left\{ \mathbf{b}^\mathsf{T}\mathbf{y} : \sum_{i=1}^{m} y_i \mathbf{a}_i = \mathbf{0}, \quad 0 \leq y_i \leq 1 \text{ for all } i \right\}, \tag{1}$$

$$\max_{\mathbf{x} \in \mathbb{R}^n,\, \mathbf{z} \in \mathbb{R}^m} \left\{ -\sum_{i=1}^{m} z_i : z_i \geq 0, \quad \mathbf{a}_i^\mathsf{T}\mathbf{x} \leq b_i + z_i \text{ for all } i \right\}. \tag{2}$$

   Prove that problem (2) is equivalent to the dual problem of (1).

2. Prove Theorem 5.6 using Corollary 5.3.

3. Prove Corollary 5.9

4. For problem (C) convex prove that the following are equivalent:

   1. $\exists \overline{\mathbf{x}} \in \mathcal{F}$ satisfying MFCQ;
   2. $\mathcal{F} \neq \emptyset$ and all points $\mathbf{x} \in \mathcal{F}$ satisfy MFCQ;
   3. $\exists \widehat{\mathbf{x}} \in \mathcal{F}$ such that $g_j(\widehat{\mathbf{x}}) < 0$ for all $j = 1, \ldots, m$.

5. Prove that LICQ implies MFCQ (using Corollary 5.3).

6. Consider problem (C) with $m = 2$, $g_1(\mathbf{x}) = \|\mathbf{x}\|_2^2 - 1$ and $g_2(\mathbf{x}) = 1 - \|\mathbf{x} - \mathbf{a}\|_2^2$, where $\mathbf{a} \in \mathbb{R}^n$ is a fixed parameter. For which parameters $\mathbf{a}$ and which feasible points $\mathbf{x}$ does LICQ not hold?