1. Consider the following variant of the $k$-means method, which we call “lazy $k$-means”: in every iteration, only one point is reassigned to a new cluster. We break ties arbitrarily.

Show that the smoothed running-time of lazy $k$-means is bounded by a polynomial in $n$ and $1/\sigma$ for $d \geq 3$. Your running-time bound may involve a factor $2^k$, but no $n^k$. However, even the factor $2^k$ can be avoided.

*Hint:* Consider epochs and $(\eta, 1)$-coarseness.

2. Fix any number $k$ of clusters and dimension $d$. Let $T(n, \sigma)$ be the smoothed number of iterations that $k$-means needs on $n$ points perturbed with standard deviation $\sigma$.

Prove that $T(n, \sigma)$ is monotonically decreasing in $\sigma$. (In particular, $T(n, \sigma) \leq T(n, 1)$ for $\sigma \geq 1$, which proves a claim from the lecture.)

3. Show that epochs of the $k$-means method have a length of at most $c$ for some constant $c$, which is independent of $d$ and $k$. 