1. Examine the program \( \max p \cdot x, x \in \{0, 1\}^n \), where \( p_1, \ldots, p_n \in \mathbb{R} \) are random variables with density upper bounded by \( \phi > 0 \). Let \( x^* \) and \( x^{**} \) denote the best and second best solutions to the above program. Show that \( \forall \varepsilon > 0, \ P[p^t x^* - p^t x^{**} \leq \varepsilon] \leq 2\varepsilon \phi n \).

2. Let \( G = (V, E) \) be an undirected graph on \( |V| = n \) vertices and non-negative edge costs \( c : E \to \mathbb{R}_{\geq 0} \) and non-negative edge lengths \( l : E \to \mathbb{R}_{\geq 0} \). Given vertices \( s, t \in V \) and a budget \( b \geq 0 \), the Constrained Shortest Path problem (CSP) is to find a minimum edge length path (i.e. \( \sum_{e \in P} l(e) \) is minimized for the path \( P \)) from \( s \) to \( t \) of cost at most \( b \).
   a) Show that when the costs and lengths are restricted to be non-negative integers, the CSP problem can be solved in time polynomial in \( n \) and \( b \).
   b) Use the above result to show that the CSP problem has polynomial smoothed complexity.