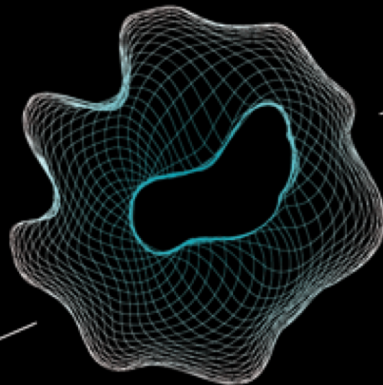


UNIVERSITY OF TWENTE.



# VERIFICATION OF CONCURRENT AND DISTRIBUTED SOFTWARE

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## OUTLINE OF THIS LECTURE

---

- How to ensure software quality?
- Classical program logic
- **VerCors exercise**
- Separation logic: reasoning about pointers
- The next challenge: concurrent software
- Permission-based separation logic
- **VerCors exercise**
- Verification of GPU kernels
- Reasoning about parallel blocks
- **VerCors exercise**
- Advanced verification features

Code voor exercises and some examples available from <https://wwwhome.ewi.utwente.nl/~marieke/VTSA>



# SOFTWARE QUALITY



Peter Naur  
1968  
Working on the  
*Software crisis*  
report

# SOFTWARE IS EVERYWHERE



All software has errors!

Software failures can have enormous impact



An important challenge: Reliable software with parallel computations



# VERIFICATION AS PART OF SOFTWARE DEVELOPMENT

```
301 * Creates a Builder by copying an existing Position instance
302 * @param other The existing instance to copy.
303
304 private Builder(EET.Position other) {
305     super(SCHEMAS);
306     if (isValidValue(fields()[0], other.delatettime)) {
307         this.delatettime = data().deepCopy(fields()[0].schema(), other.delatettime);
308         fieldSetFlags()[0] = true;
309     }
310     if (isValidValue(fields()[1], other.granularity)) {
311         this.granularity = data().deepCopy(fields()[1].schema(), other.granularity);
312         fieldSetFlags()[1] = true;
313     }
314     if (isValidValue(fields()[2], other.granularity_unit)) {
315         this.granularity_unit = data().deepCopy(fields()[2].schema(), other.granularity_unit);
316         fieldSetFlags()[2] = true;
317     }
318     if (isValidValue(fields()[3], other.segment_id)) {
319         this.segment_id = data().deepCopy(fields()[3].schema(), other.segment_id);
320         fieldSetFlags()[3] = true;
321     }
322     if (isValidValue(fields()[4], other.source)) {
323         this.source = data().deepCopy(fields()[4].schema(), other.source);
324         fieldSetFlags()[4] = true;
325     }
326     if (isValidValue(fields()[5], other.type)) {
327         this.type = data().deepCopy(fields()[5].schema(), other.type);
328         fieldSetFlags()[5] = true;
329     }
330     if (isValidValue(fields()[6], other.quantity)) {
331         this.quantity = data().deepCopy(fields()[6].schema(), other.quantity);
332         fieldSetFlags()[6] = true;
333     }
334     if (isValidValue(fields()[7], other.quantity_unit)) {
335         this.quantity_unit = data().deepCopy(fields()[7].schema(), other.quantity_unit);
336         fieldSetFlags()[7] = true;
337     }
338 }
```

Warning: this method does not have intended behaviour.  
Click [here](#) for a counterexample

Warning: at this point a nullpointer exception can occur.  
Click [here](#) for an example execution

Realising this dream requires substantial research

- Enlarge class of properties that can be established
- Automatic feedback

Dates back  
to the 60-ies

# SPECIFYING PROGRAM BEHAVIOUR

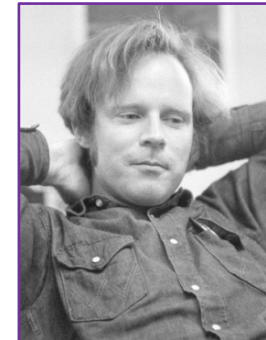
Use logic to describe behaviour of program components

- **Precondition:** what do you know in advance?
- **Postcondition:** what holds afterwards

Example: `increaseBy(int n)`

requires  $n \geq 0$

ensures  $x == \text{old}(x) + n$



Bob Floyd

Hoare triples + logic  
Notation:  $\{P\}S\{Q\}$   
Syntactic verification of programs

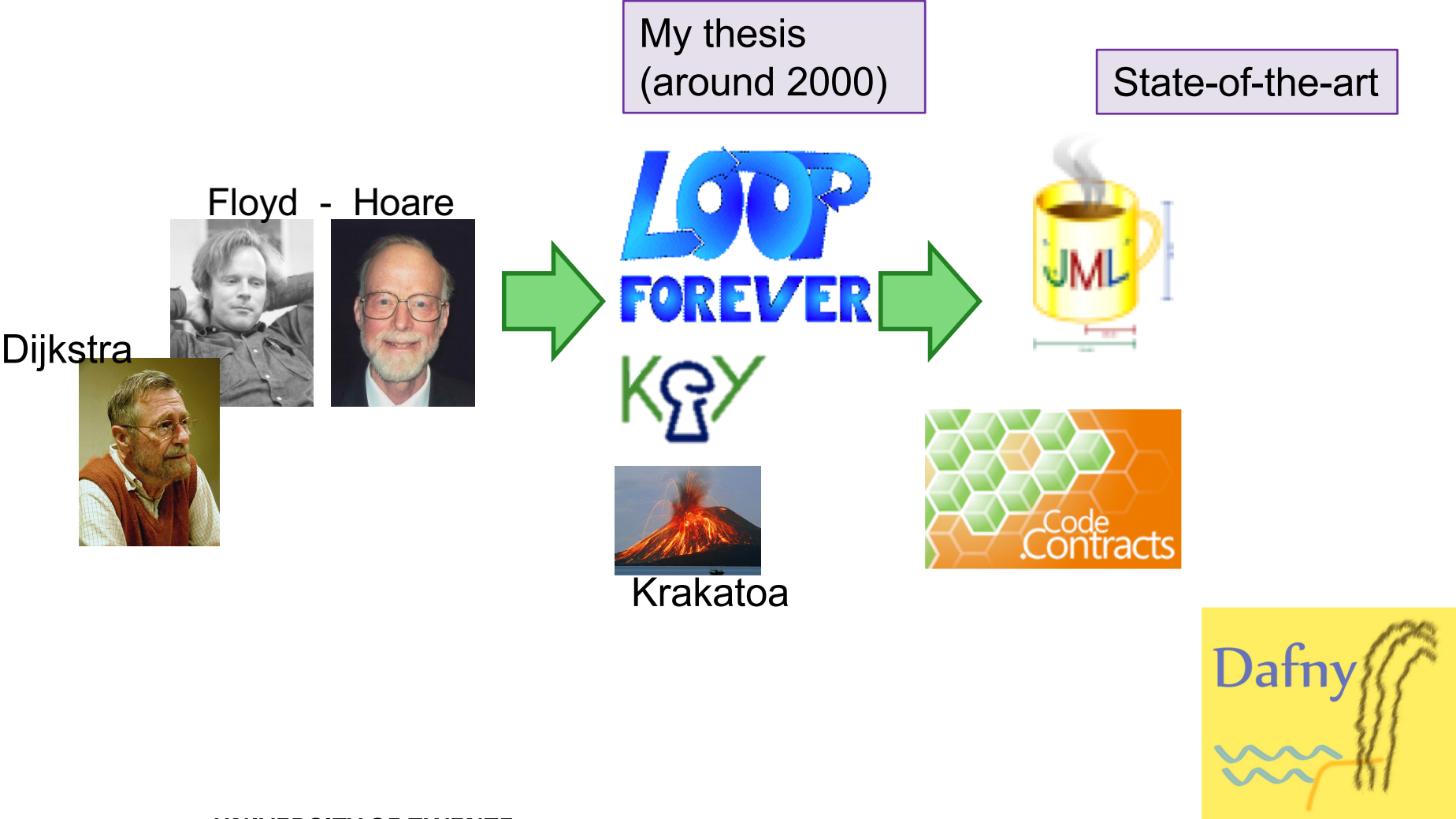


Tony Hoare

precondition

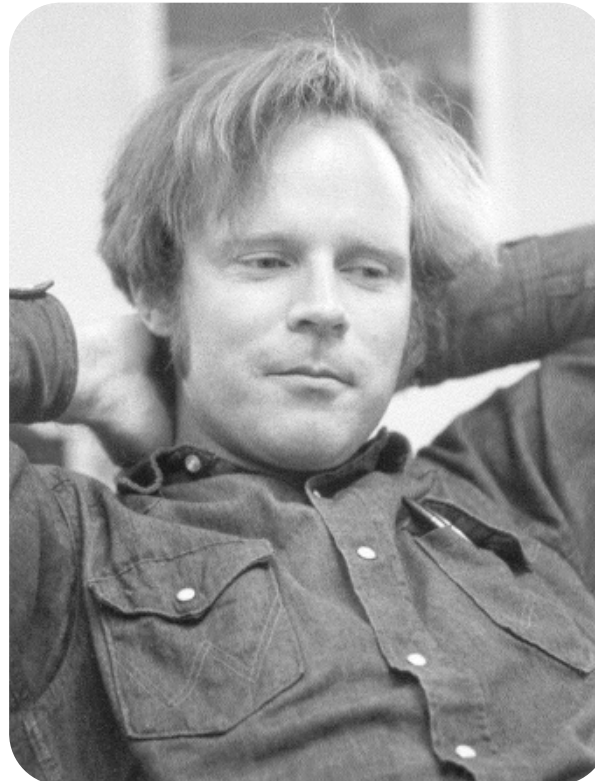
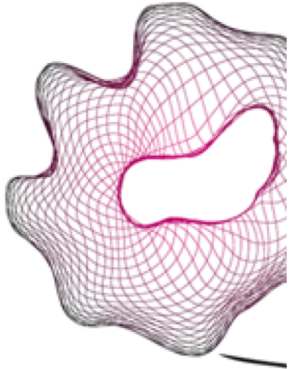
postcondition

# HISTORY OF PROGRAM VERIFICATION



# PROGRAM LOGIC

---



Bob Floyd  
1936 - 2001



# PRE- AND POSTCONDITIONS

---

- **Precondition:** property that should be satisfied when method is called – otherwise correct functioning of method is not guaranteed
- **Postcondition:** property that method establishes – caller can assume this upon return of method
- Method specification is contract between implementer and caller of method.
  - Caller promises to call method only in states in which precondition holds
  - Implementer guarantees postcondition will be established



# HOARE TRIPLES

---

- $\{P\}S\{Q\}$



- Due to Tony Hoare (1969)

- Meaning: if  $P$  holds in initial state  $s$ , and execution of  $S$  in  $s$  terminates in state  $s'$ , then  $Q$  holds in  $s'$

- Formally:

$$\{P\}S\{Q\} = \forall s. P(s) \wedge (S, s) \rightarrow s' \Rightarrow Q(s')$$

# HOARE LOGIC

---

- Hoare triples: specify behaviour of methods
- How to guarantee that methods indeed respect this behaviour?
  
- Collection of derivation rules to reason about Hoare triples
- Rules defined by induction on the program structure
- Proven sound w.r.t. program semantics
  
- Here: a very simple language, but exists for more complicated languages

# AXIOMS

---

$$\text{Skip} \frac{}{\{P\}\text{Skip}\{P\}}$$

$$\text{Ass.} \frac{}{\{P[v:= e]\}v := e\{P\}}$$

# STATEMENT DECOMPOSITION

---

$$\text{Seq} \frac{\{P\}S1\{Q\} \quad \{Q\}S2\{R\}}{\{P\}S1;S2\{R\}}$$

$$\text{If} \frac{\{P \wedge b\}S1\{Q\} \quad \{P \wedge \neg b\}S2\{Q\}}{\{P\}\text{if } (b) \text{ } S1 \text{ else } S2 \{Q\}}$$

(\*): precondition strengthening

# EXAMPLE

$$a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1 [z := 1] = a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge 1 = 1$$

$$\text{Ass} \frac{\{a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge 1 = 1\} z := 1 \{a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1\}}{a \geq 0 \wedge n \geq 0 \wedge k = 0 \Rightarrow a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge 1 = 1}$$

$$a \geq 0 \wedge n \geq 0 \wedge k = 0 [k := 0] = a \geq 0 \wedge n \geq 0 \wedge 0 = 0$$

$$\text{Ass} \frac{\{a \geq 0 \wedge n \geq 0 \wedge 0 = 0\} k := 0 \{a \geq 0 \wedge n \geq 0 \wedge k = 0\}}{a \geq 0 \wedge n \geq 0 \Rightarrow a \geq 0 \wedge n \geq 0 \wedge 0 = 0} (*)$$

$$\{a \geq 0 \wedge n \geq 0\} k := 0 \{a \geq 0 \wedge n \geq 0 \wedge k = 0\} \quad (*)$$

$$\text{Seq} \frac{\{a \geq 0 \wedge n \geq 0 \wedge k = 0\} z := 1 \{a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1\}}{\{a \geq 0 \wedge n \geq 0\} k := 0; z := 1 \{a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1\}}$$

$$\text{Seq} \frac{\{a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1\} \text{ while } (k < n) \{z := z * a; k := k + 1;\} \{z = a^n\}}{\{a \geq 0 \wedge n \geq 0\} k := 0; z := 1; \text{ while } (k < n) \{z := z * a; k := k + 1;\} \{z = a^n\}}$$

# RULES OF CONSEQUENCE

---

$$\text{Pre. Str.} \frac{P \Rightarrow P' \quad \{P'\}S\{Q\}}{\{P\}S\{Q\}}$$

$$\text{Post. Weak.} \frac{\{P\}S\{Q\} \quad Q \Rightarrow Q'}{\{P\}S\{Q'\}}$$

# LOOPS

---

$$\text{Loop} \frac{\{I \wedge b\}S\{I\}}{\{I\}\text{while } (b) S \{I \wedge \neg b\}}$$

- $I$  called **loop invariant**
- Preserved by every iteration of the loop
- Can in general not be found automatically



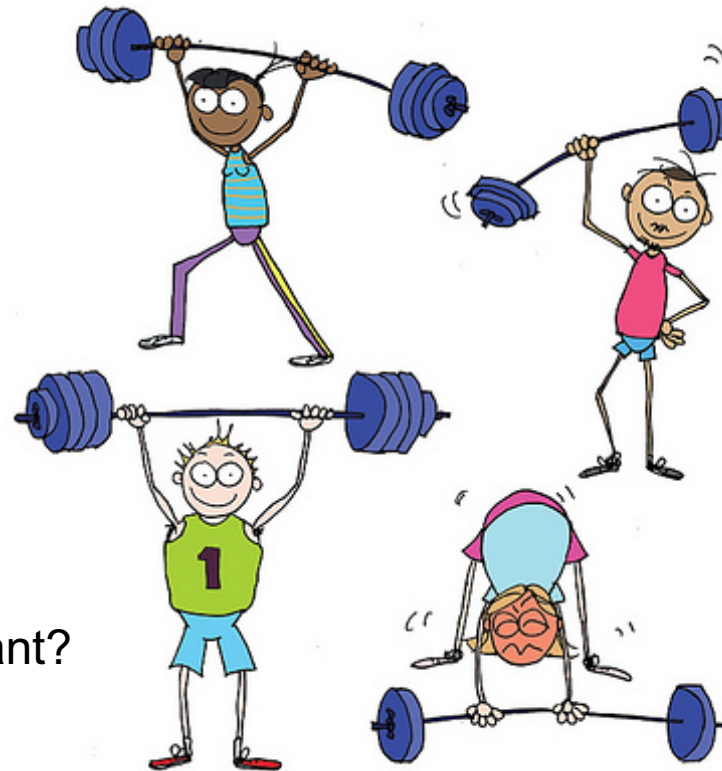
# EXAMPLE: METHOD POWER

---

```
{ a ≥ 0 ∧ n ≥ 0 }  
k := 0;  
z := 1;  
{ a ≥ 0 ∧ n ≥ 0 ∧ k = 0 ∧ z = 1 }  
while (k < n)  
  { z := z * a;  
    k := k + 1;  
  }  
{ z = a^n }
```

What should be the loop invariant?

$z = a^k \wedge k \leq n \wedge a \geq 0 \wedge k \geq 0$



## EXAMPLE CONTINUED

---

$$\text{Ass } \frac{}{\{z^*a = a^{(k+1)} \wedge k + 1 \leq n \wedge a \geq 0\} z := z * a \{z = a^{(k+1)} \wedge k + 1 \leq n \wedge a \geq 0\}}$$

$$z = a^k \wedge k \leq n \wedge a \geq 0 \wedge !(k = n) \Rightarrow z^*a = a^{(k+1)} \wedge a \geq 0 \wedge k + 1 \leq n$$

Pre. Str.

$$\text{Ass } \frac{}{\{z = a^{(k+1)} \wedge k + 1 \leq n \wedge a \geq 0\} k := k + 1 \{z = a^k \wedge k \leq n \wedge a \geq 0\}}$$

Seq

$$\frac{\{z = a^k \wedge k \leq n \wedge a \geq 0 \wedge !(k = n)\} z := z * a \{z = a^{(k+1)} \wedge k + 1 \leq n \wedge a \geq 0\}}{\{z = a^k \wedge k \leq n \wedge a \geq 0 \wedge !(k = n)\} z := z * a; k := k + 1 \{z = a^k \wedge k \leq n \wedge a \geq 0\}}$$

Loop

$$\frac{\{z = a^k \wedge k \leq n \wedge a \geq 0 \wedge !(k = n)\} z := z * a; k := k + 1 \{z = a^k \wedge k \leq n \wedge a \geq 0\}}{\{z = a^k \wedge k \leq n \wedge a \geq 0\} \text{ while } (!(k = n)) \{z := z * a; k := k + 1;\} \{z = a^k \wedge k \leq n \wedge a \geq 0 \wedge k = n\}} \\ z = a^k \wedge k \leq n \wedge a \geq 0 \wedge k = n \Rightarrow z = a^n$$

Post. Weak.

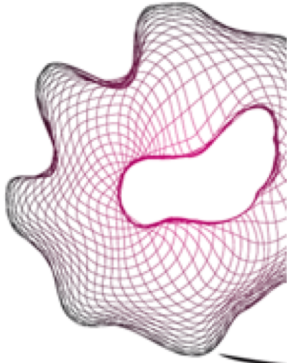
$$\frac{}{\{z = a^k \wedge k \leq n \wedge a \geq 0\} \text{ while } (!(k = n)) \{z := z * a; k := k + 1;\} \{z = a^n\}}$$

$$a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1 \Rightarrow z = a^k \wedge k \leq n \wedge a \geq 0$$

Pre. Str.

$$\frac{}{\{a \geq 0 \wedge n \geq 0 \wedge k = 0 \wedge z = 1\} \text{ while } (!(k = n)) \{z := z * a; k := k + 1;\} \{z = a^n\}}$$

# TOOL SUPPORT FOR PROGRAM VERIFICATION



Rustan Leino

# A CALCULATIONAL APPROACH

---

Many intermediate predicates can be computed

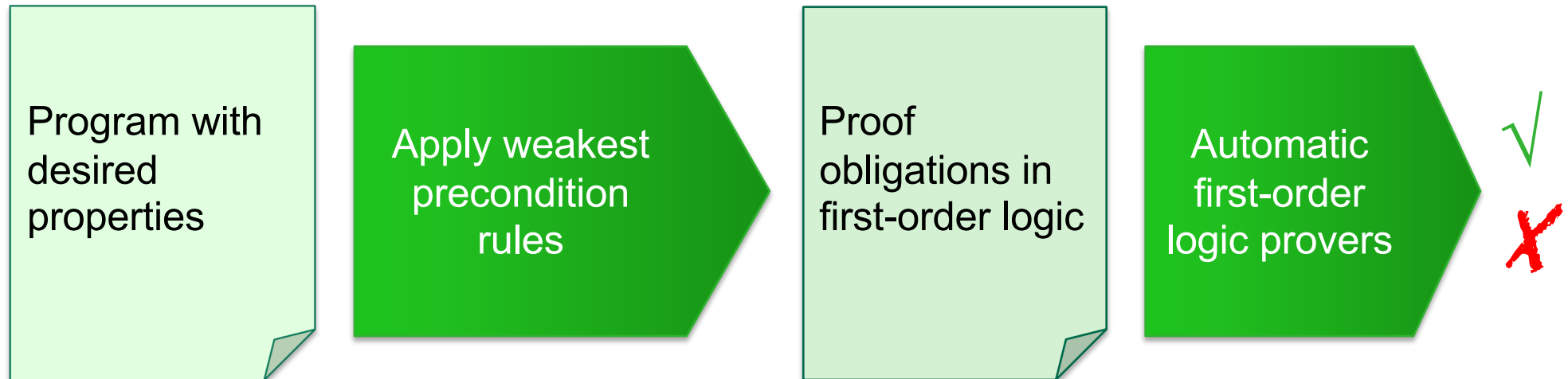
- Weakest liberal precondition  $wp(S, Q)$
- The weakest predicate such that  $\{wp(S, Q)\}S\{Q\}$
- Due to Edsger Dijkstra (1975)
- Calculus allows to compute weakest preconditions of sequential code
- Proof obligations: preconditions imply weakest liberal preconditions
- Loop invariants still given explicitly



1932 -  
2002

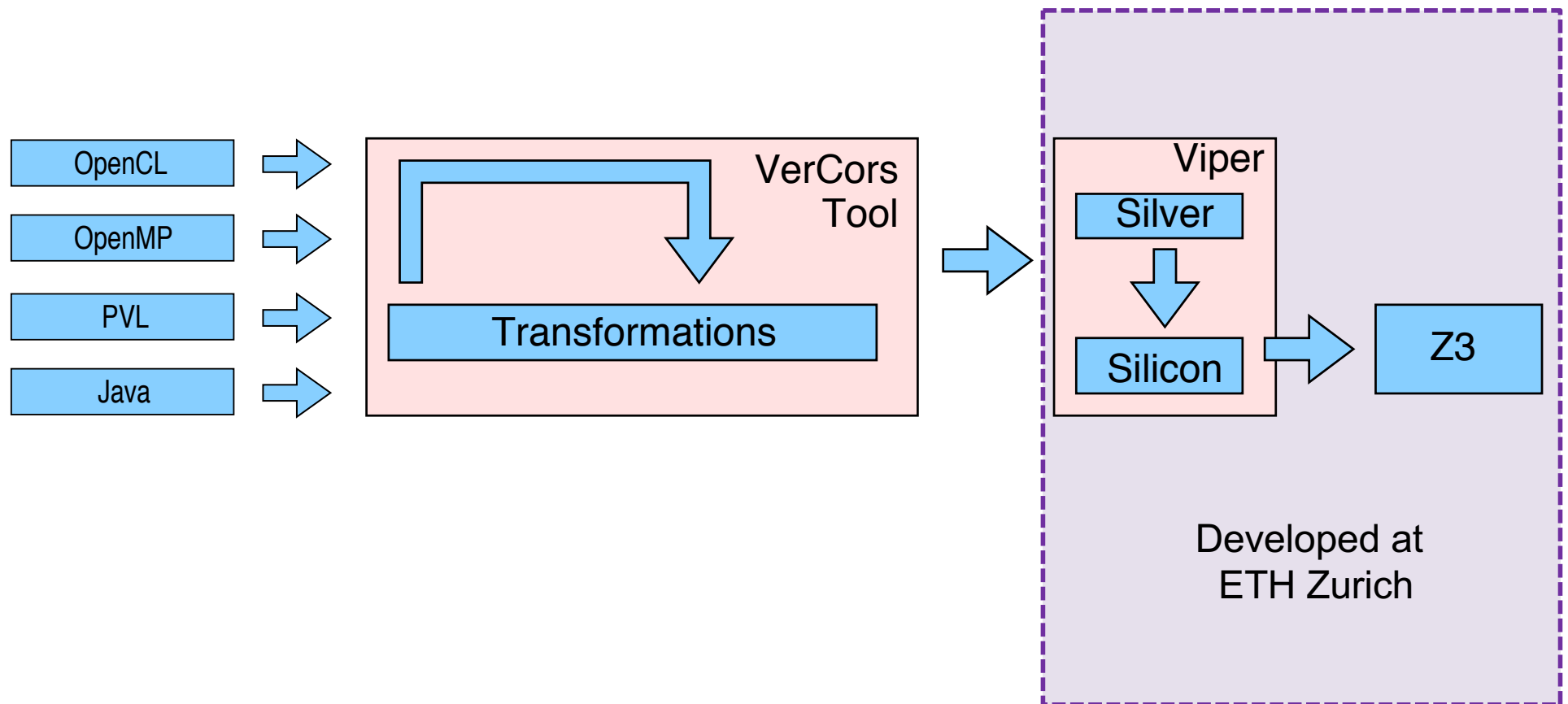
# AUTOMATION

---



Preferably also **counter example**: why does program not have desired behaviour

# VERCORS TOOL ARCHITECTURE



See iFM 2017

# PROGRAM CORRECTNESS IN VERCORS

---

- PVL syntax: <https://github.com/utwente-fmt/vercors/wiki/PVL-Syntax>
- Two kinds of verification:
  - Memory safety (postcondition true), method will terminate without exceptions
  - Functional correctness: postcondition expresses something about poststate of the method
- Two useful abbreviations
  - Context: pre- and postcondition
  - Invariant: pre- and postcondition, and at loop entry and exit



# EXERCISES

Code voor exercises and some examples available from  
<https://wwwhome.ewi.utwente.nl/~marieke/VTSA>



# LIMITATIONS OF CLASSICAL PROGRAM LOGIC

---

- Idealised language
- No side-effects in conditions
- No pointers
- No multi-threading

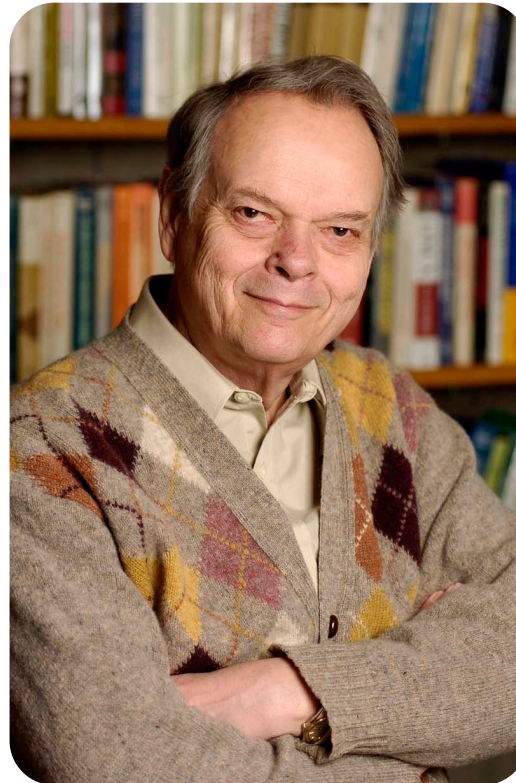
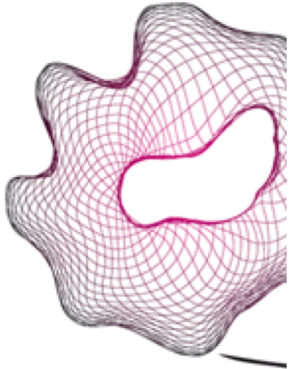
## Separation logic

- Reasoning about pointers
- Natural extension to multi-threading



# SEPARATION LOGIC

---



John Reynolds  
1935 - 2013

# THE CHALLENGE OF POINTER PROGRAMS

---

```
class C {  
  
    D f;  
    D g;  
}
```

```
class D {  
    int x;  
    D() {  
        x = 0;  
    }  
}
```

```
ensures c.g.x == 0;
```

```
void m(C c) {  
    d = new D;  
    c.f = d;  
    c.g = d;  
    update_x(c.f, 3);  
}
```

```
ensures d.x == v;
```

```
void update_x(D d, int v) {  
    d.x = v;  
}  
}
```

This should **not** be verified!

# SEPARATION LOGIC

---

- State distinguishes heap and store
- Heap contains dynamically allocated data that exists during run-time of program  
(Object-oriented program: the objects are stored on the heap)
- Store (or call stack) contains data related to method call (parameters, local variables)
- Heap accessed by pointers
- Locations on heap can be aliased
- Main idea: assertions about state can be decomposed into assertions about **disjoint substates**

# INTUITIONISTIC SEPARATION LOGIC

---

Syntax extension of predicate logic:

$$\varphi ::= e.f \rightarrow e' \mid \varphi * \varphi \mid \varphi - * \varphi \mid \dots$$

where  $e$  is an expression, and  $f$  a field

Meaning:

- $e.f \rightarrow e'$  – heap contains location pointed to by  $e.f$ , containing the value given by the meaning  $e'$
- $\varphi_1 * \varphi_2$  – heap can be split in disjoint parts, satisfying  $\varphi_1$  and  $\varphi_2$ , respectively
- $\varphi_1 - * \varphi_2$  – if heap extended with part that satisfies  $\varphi_1$ , composition satisfies  $\varphi_2$

Monotone w.r.t. extensions of the heap

Magic wand  
not frequently  
used

# ADVANTAGES OF SEPARATION LOGIC

---

- Reasoning about programs with pointers
- Two interpretations  $e.f \rightarrow v$ 
  - Field  $e.f$  contains value  $v$
  - Permission to access field  $e.f$ 

A field can only be accessed or written if  $e.f \rightarrow \_$  holds!
- Implicit disjointness of parts of the heap allows reasoning about (absence) of **aliasing**

$x.f \rightarrow \_ * y.f \rightarrow \_$  implicitly says that  $x$  and  $y$  are **not aliases**
- Local reasoning
  - only reason about heap that is actually accessed by code fragment
  - rest of heap is implicitly unaffected: **frame rule**

# PROOF RULE FOR UPDATES OF THE HEAP

---

---

$$\{e.f \rightarrow \_ \} e.f = v \{e.f \rightarrow v\}$$

- For simplicity  $v$  is typically assumed to be a simple (unqualified) expression
- Any assignment  $e.f = e'.g$  can be split up in  $x = e'.g; e.f = x$

# EXAMPLE: CLASS BOX

---



```
class Box {  
  int cnts;
```

```
  requires this.cnts → _;  
  ensures this.cnts → 0;  
  void set (int o) {  
    this.cnts = o;  
    return null;  
  }
```

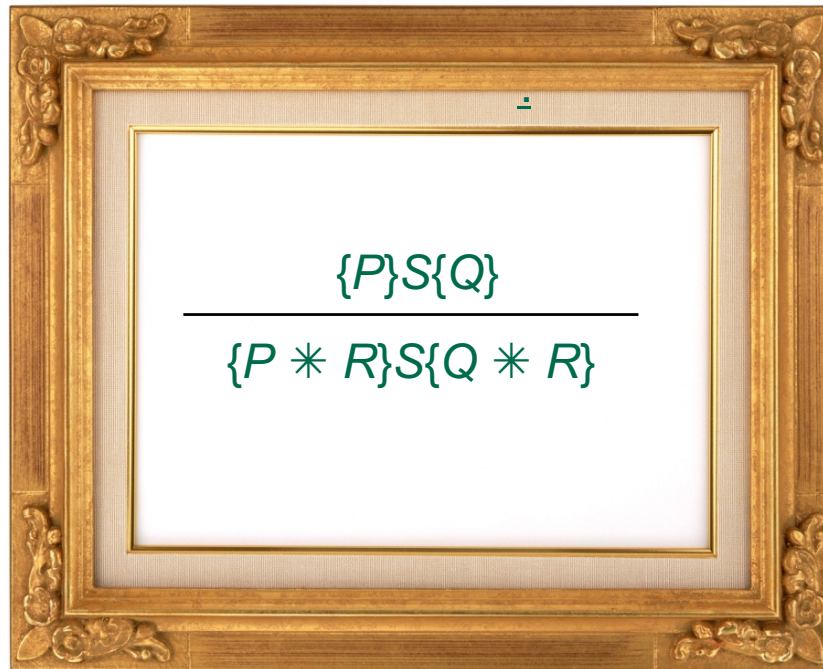
```
  requires this.cnts → X;  
  ensures this.cnts → X ∧ result = X;  
  int get() {  
    return this.cnts;  
  }
```

```
}  
  
} Compare with specifications  
in classical Hoare logic  
requires true;  
ensures this.cnts == 0;
```



# FRAME RULE

---



where  $R$  does not contain any variable that is modified by  $S$ .

# THE CHALLENGE OF POINTER PROGRAMS

---

```
class C {  
  D f;  
  D g;  
}
```

```
class D {  
  int x;
```

```
  D() {  
    x = 0;  
  }
```

```
ensures c.g.x == 0;
```

```
void m(C c) {
```

```
  d = new D;
```

```
  c.f = d;
```

```
  c.g = d;
```

```
  update_x(c.f, 3);
```

```
}
```

$c.f \rightarrow \_ * c.g \rightarrow \_$   
does not hold

Empty frame

```
ensures d.x == v;
```

```
void update_x(D d, int v) {
```

```
  d.x = v;
```

```
}
```

# SEPARATION LOGIC VS IMPLICIT DYNAMIC FRAMES

---

- Classical separation logic:  $\text{this.cnts} \rightarrow X$
- Implicit dynamic frames:  $\text{Perm}(\text{this.cnts}) * \text{this.cnts} == X$
- VerCors:  $\text{Perm}(\text{this.cnts}, \text{write}) ** \text{this.cnts} == X$

# BOX IN VERCORS

---

```
class Box {  
    int cnts;  
  
    requires Perm(this.cnts, write);  
    ensures Perm(this.cnts, write);  
    void setCnts (int o) {  
        this.cnts = o;  
    }  
}  
  
given int x;  
requires Perm(this.cnts, write) **  
    this.cnts == x;  
ensures Perm(this.cnts, write) **  
    \result == x;  
int getCnts () {  
    return this.cnts;  
}  
}
```

Given: ghost parameter



# CONCURRENCY: THE NEXT CHALLENGE

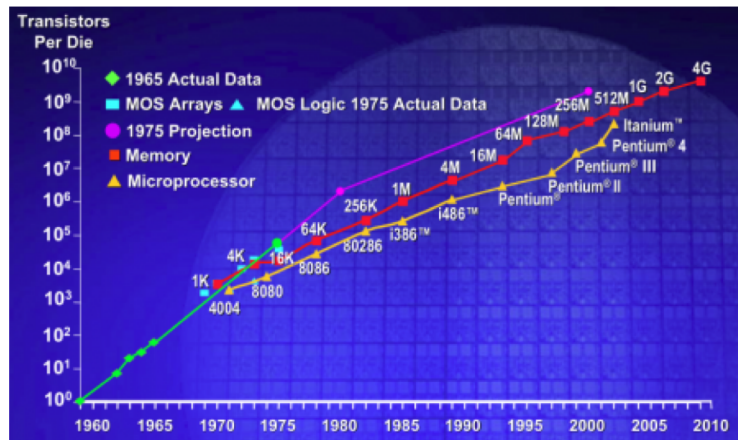
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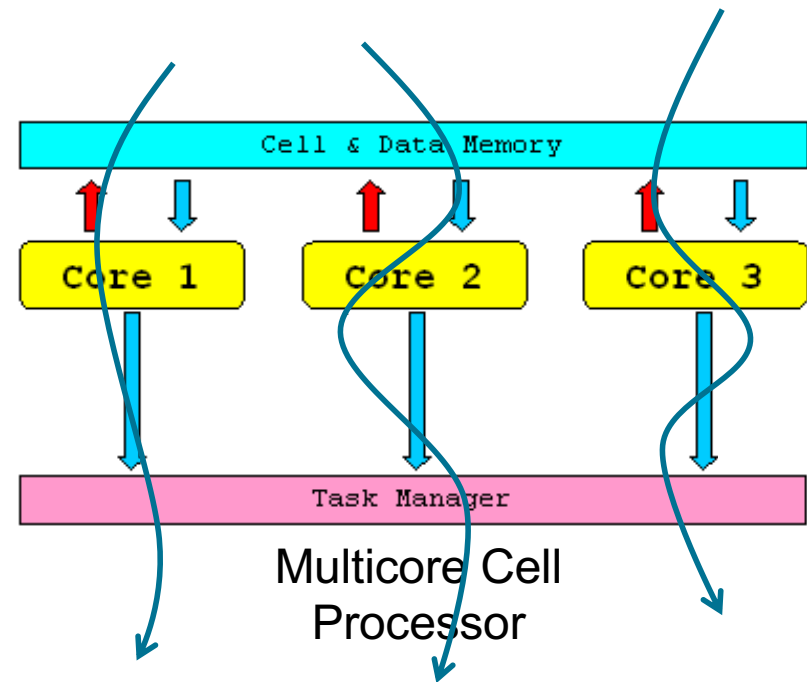
Doug Lea

# THE FUTURE OF COMPUTING IS MULTICORE

Single core processors:  
The end of Moore's law



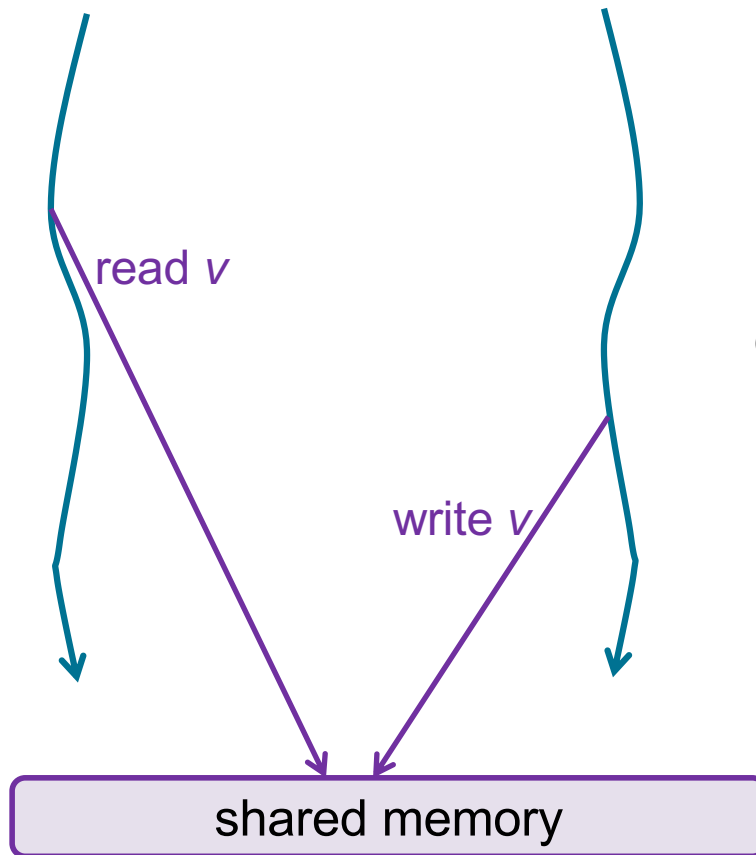
Solution:  
Multi-core processors



Multiple threads of execution

Coordination problem shifts  
from hardware to software

# MULTIPLE THREADS CAUSE PROBLEMS



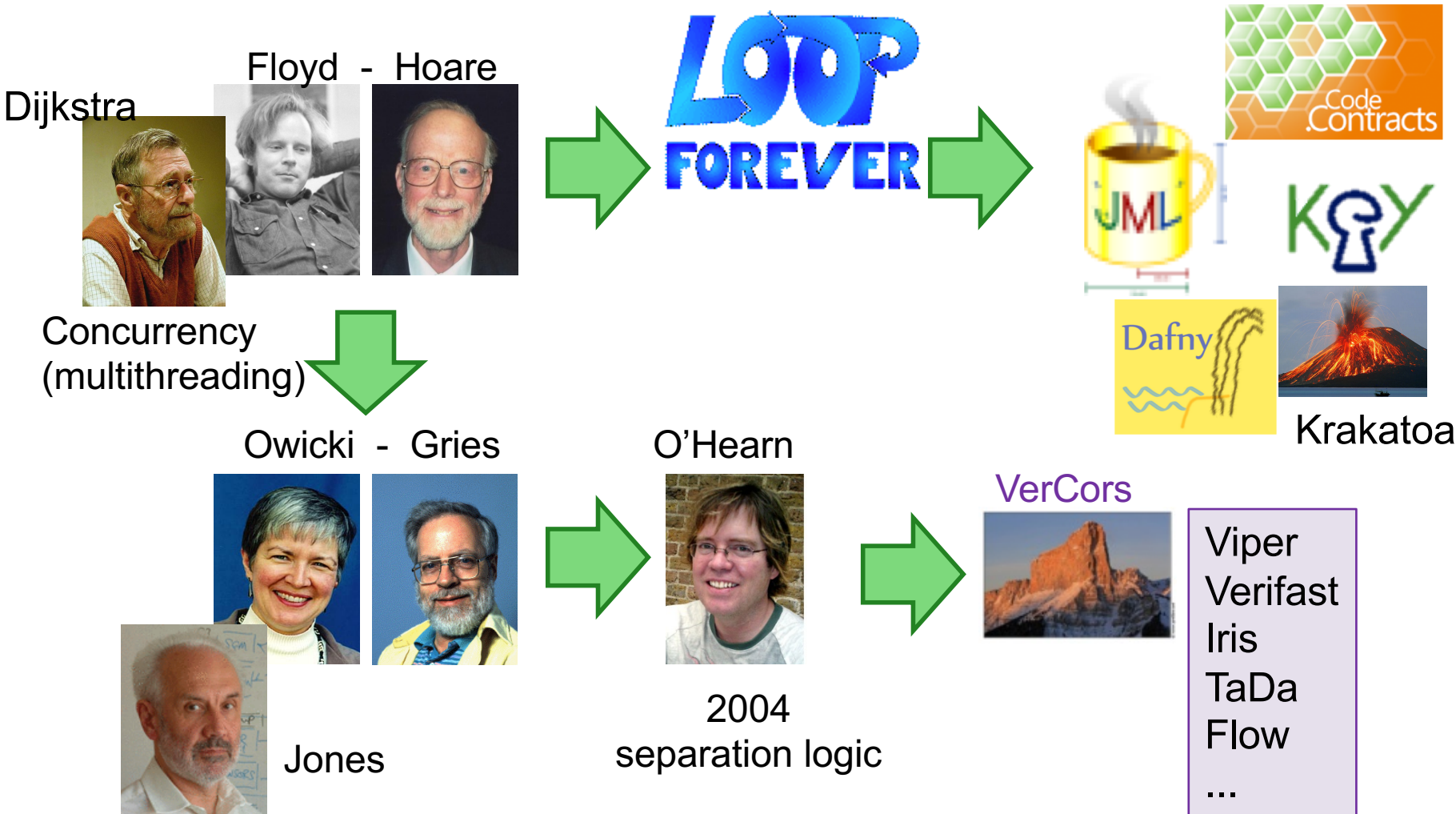
- Order?
- More threads?



Possible consequences:  
errors such as data races caused  
lethal bugs as in Therac-25



# VERIFICATION OF MULTITHREADED PROGRAMS





# SPECIFICATIONS IN A CONCURRENT SETTING

---

requires true

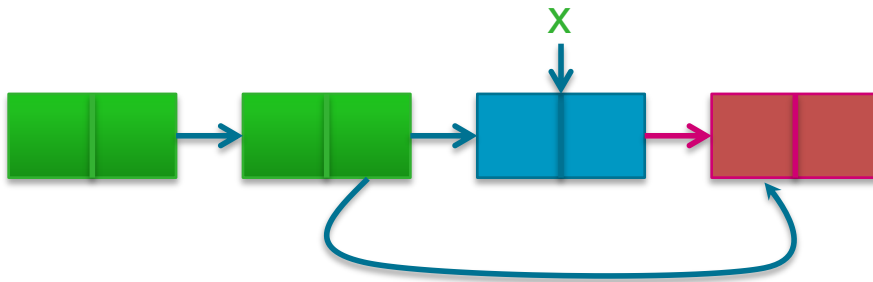
ensures  $x$  is the last element in the list

```
void addToList(Elem x) {  
    // code  
}
```

Any other thread  
might invalidate  
this!

' $x$  is in the list'  
cannot even be  
guaranteed!

Except when no  
other thread can  
update the list





## SOME HISTORY: REASONING ABOUT THREADS

---



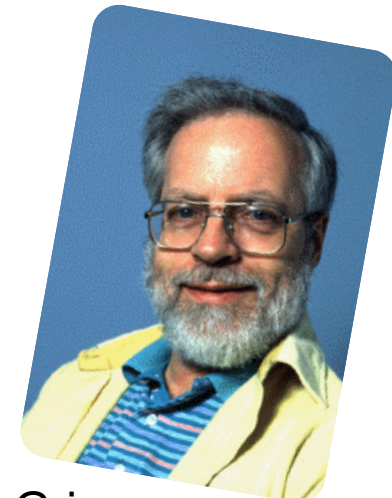
Susan Owicki



# OWICKI-GRIES METHOD (1975)

---

- For each thread: give a **complete** proof outline
- Verify each thread w.r.t. the proof outline
- For each annotation in the proof outline, show that it cannot be invalidated by any other thread: **interference freedom**



David Gries

# RELY-GUARANTEE METHOD

---

- Jones (1980)
- Compositional
- For each thread, specify
  - what it assumes from other threads
  - what it guarantees to other threads



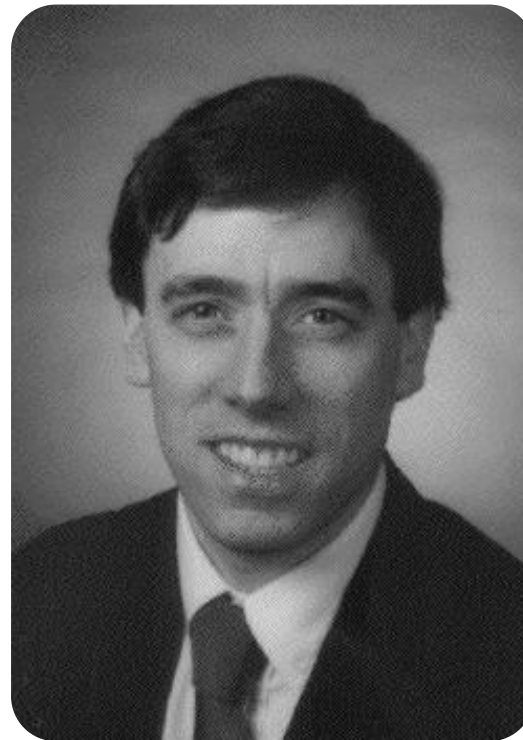
**Rely:** what transitions may other threads make  
**Guarantee:** what transitions may current thread make

$$\begin{aligned} & \text{rely} \vee \text{guar1} \Rightarrow \text{rely2} \\ & \text{rely} \vee \text{guar2} \Rightarrow \text{rely1} \\ & \text{guar1} \vee \text{guar2} \Rightarrow \text{guar} \\ \hline & \langle \text{rely}_i, \text{guar}_i \rangle : \{P_i\} S_i \{Q_i\}, i = 1, 2 \\ \hline & \langle \text{rely}, \text{guar} \rangle : \{P\} S1 \parallel S2 \{Q\} \end{aligned}$$



# CONCURRENT SEPARATION LOGIC

---

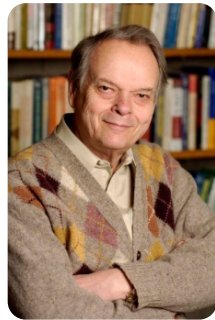


John Boyland



# JOHN REYNOLDS'S 70TH BIRTHDAY PRESENT

---



---

$\{P_1\}S_1\{Q_1\} \dots \dots \dots \{P_n\}S_n\{Q_n\}$

$\{P_1 * \dots * P_n\} S_1 \parallel \dots \parallel S_n \{Q_1 * \dots * Q_n\}$

where no variable free in  $P_i$  or  $Q_i$  is changed in  $S_j$  (if  $i \neq j$ )

# EXAMPLE

---



$$\frac{}{\{x = 0\} x := x + 1; x := x + 1 \{x = 2\}}$$



$$\frac{}{\{y = 0\} y := y + 1; y := y + 1 \{y = 2\}}$$

$$\frac{}{\{x = 0 * y = 0\} x := x + 1; x := x + 1 \parallel y := y + 1; y := y + 1 \{x = 2 * y = 2\}}$$

No interference between the threads

# WHY IS THIS NOT SUFFICIENT?

---

- Simultaneous reads not allowed

1. Distinguish between read and write accesses

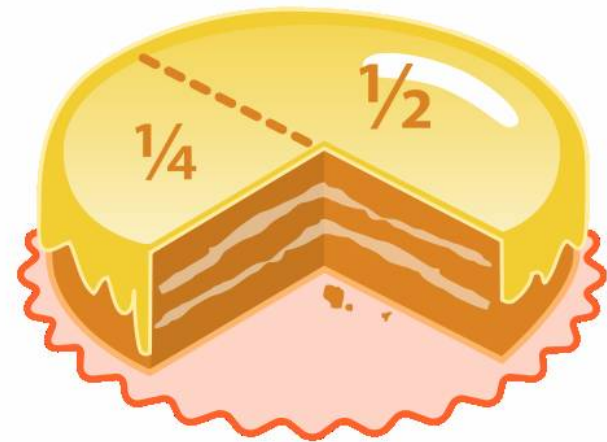
- Number of parallel threads is fixed



# PERMISSIONS

---

- **Permission** to access a variable
- Value between 0 and 1
- Full permission **1** allows to change the variable
- Fractional permission in  $(0, 1)$  allows to inspect a variable
- Points-to predicate decorated with a permission
- Global invariant: for each variable, the sum of all the permissions in the system is never more than 1
- Permissions can be split and combined



Permissions on n equally distributed over threads

## EXAMPLE



---

 $\{PointsTo(x,1,0) * Perm(n, \frac{1}{2})\}$  $x := x + n; x := x + n$ 

---

 $\{PointsTo(x,1,2*n) * Perm(n, \frac{1}{2})\}$ 

---

 $\{PointsTo(y,1,0) * Perm(n, \frac{1}{2})\}$  $y := y + n; y := y + n$ 

---

 $\{PointsTo(y,1,2*n) * Perm(n, \frac{1}{2})\}$ 

---

 $\{PointsTo(x,1,0) * PointsTo(y,1,0) * Perm(n,1)\}$  $x := x + n; x := x + n \parallel y := y + n; y := y + n$ 

---

 $\{PointsTo(x,1,2*n) * PointsTo(y,1,2*n) * Perm(n,1)\}$ 

$Perm(x,1) = Perm(x, \frac{1}{2}) * Perm(x, \frac{1}{2})$

Shared variable is only read  
No interference between the threads

# WHY IS THIS NOT SUFFICIENT?

---

- Simultaneous reads not allowed

1. Distinguish between read and write accesses

- Number of parallel threads is fixed

2. Dynamic thread creation

Thread specifications indicate how permissions should be distributed

# EXAMPLE

```
class List {  
    int val; List next;  
    ...  
}
```

```
class T {  
    List y;  
    void run() { ... }  
}
```

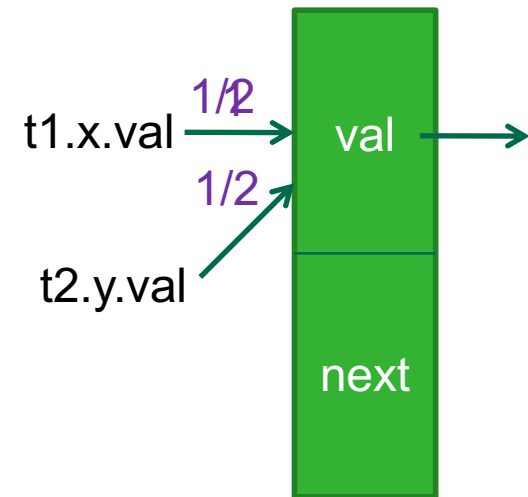
t1

```
x := new List;  
x.val := ...;  
t2 := new T;  
t2.y := x;  
fork t2;  
read x.val;  
...
```

```
join t2;  
x.val := ...;
```

t2

```
run(){  
    ...  
    read y.val  
    ...  
}
```



## SPECIFICATION FOR RUN METHOD IN T2

---

```
requires Perm (y.val, 1/2);  
ensures Perm(y.val, 1/2);  
void run() {...}
```

- Forking thread has to give up required permissions
- Joining thread gains back ensured permissions

What happens if `run` is specified as follows:

```
requires Perm(y.val, 1);  
ensures Perm(y.val, 1);;  
void run() {...}
```

# EXAMPLE

```
class List {  
    int val; List next;  
    ...  
}  
  
class T {  
    List y;  
    void run() { ... }  
}
```

t1

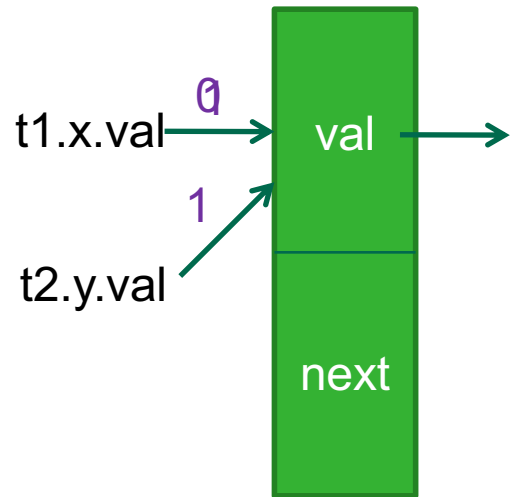
```
x := new List;  
x.val := ...;  
t2 := new T;  
t2.y := x;  
fork t2();  
read x.val;  
...
```

t2

```
run(){  
    ...  
    read y.val t1.x.val  
    ...  
}
```

NOT ALLOWED!

Now the permissions are back



# RESOURCE INVARIANT – CLASSICAL APPROACH



- Lock  $x$  acquired and released with `lock x` and `unlock x`
- Each lock has associated resource invariant
- Lock acquired  $\longrightarrow$  resource invariant lend to thread
- Lock released  $\longrightarrow$  resource invariant taken back from thread
- Class Object contains predicate  
`resource lock_invariant() = true;`
- In rules: if  $I$  is resource invariant of  $x$   
`{true} lock x {I}`  
`{I}unlock x{true}`
- This is sound only for single-entrant locks

```
{true}  
lock x;  
{I}  
lock x;  
{I * I}  
...
```

Resource  $I$  has  
been duplicated!

# LOCKS IN PVL

---

```
class locktest {  
  
  resource lock_invariant() =  
    Perm(x, 1/2);
```

```
  int x;
```

```
  locktest(int y) {  
    x = y;  
  }
```

```
  void m() {  
    lock this;  
    int v = x;  
    unlock this;  
  }  
}
```

Perm(x, write) = Perm(x, 1)  
Perm(x, read) = Perm(x, v) for some v





---

# EXERCISES

Code voor exercises and some examples available from  
<https://wwwhome.ewi.utwente.nl/~marieke/VTSA>