Basic Math Formulas

**Arithmetic operations** \((ab)\text{ means }a \times b\)

<table>
<thead>
<tr>
<th>Operation</th>
<th>Formula</th>
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</thead>
<tbody>
<tr>
<td>(a(b + c))</td>
<td>(ab + ac)</td>
<td>(a + \frac{c}{d} = \frac{ad + bc}{bd})</td>
<td>(\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd})</td>
</tr>
</tbody>
</table>

**Logarithm:** \(\log_a x = y \iff a^y = x\)

\[
\log_a (xy) = \log_a (x) + \log_a (y) \\
\log_a \left(\frac{x}{y}\right) = \log_a (x) - \log_a (y)
\]

**Factors:** \((x + a)(x + b) = x^2 + (a + b)x + ab\)

\((x + a)(x - a) = x^2 - a^2\)

\[ax^2 + bx + c = 0 \implies x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

**Absolute Value (for \(a > 0\)):**

\[|x| = a \implies x = a \quad \text{or} \quad x = -a\]

\[|x| < a \implies -a < x < a\]

\[|x| > a \implies x > a \quad \text{or} \quad x < -a\]

**Right triangle**

**Pythagorean Theorem:** \(a^2 + b^2 = c^2\)

\[
sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{a}{c}\]

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}
\]

**Important angles**

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(\theta) rad (\frac{\theta}{180}\pi)</th>
<th>(\sin \theta)</th>
<th>(\cos \theta)</th>
<th>(\tan \theta = \frac{\sin \theta}{\cos \theta})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>(\frac{\pi}{6})</td>
<td>(\frac{1}{2})</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{1}{\sqrt{3}})</td>
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<tr>
<td>45°</td>
<td>(\frac{\pi}{4})</td>
<td>(\frac{\sqrt{2}}{2})</td>
<td>(\frac{\sqrt{2}}{2})</td>
<td>1</td>
</tr>
<tr>
<td>60°</td>
<td>(\frac{\pi}{3})</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{1}{2})</td>
<td>(\sqrt{3})</td>
</tr>
<tr>
<td>90°</td>
<td>(\frac{\pi}{2})</td>
<td>1</td>
<td>0</td>
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</table>

**Trigonometric properties**

\[
\sin^2(x) + \cos^2(x) = 1 \\
\sin(-x) = -\sin(x) \\
\cos(-x) = \cos(x) \\
\tan(-x) = -\tan(x) \\
\sin(\frac{\pi}{2} - x) = \cos(x) \\
\cos(\frac{\pi}{2} - x) = \sin(x) \\
\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y) \\
\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y) \\
\sin(2x) = 2\sin(x)\cos(x) \\
\cos(2x) = 1 - 2\sin^2(x) \\
\tan(x) = \frac{\sin(x)}{\cos(x)}
\]

**Powers and roots**

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<tr>
<th>Formula</th>
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<tbody>
<tr>
<td>(x^a x^b = x^{a+b})</td>
<td>((x^a)^b = x^{ab})</td>
<td>(\frac{x^a}{x^b} = x^{a-b})</td>
</tr>
<tr>
<td>((xy)^a = x^a y^a)</td>
<td>(x^{-a} = \frac{1}{x^a})</td>
<td>(\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}})</td>
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</table>

**Natural logarithm:** \(\ln x = \log_e x \quad (e \approx 2.71828..)\)

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<thead>
<tr>
<th>Formula</th>
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<th>Formula</th>
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<tbody>
<tr>
<td>(\ln (xy) = \ln (x) + \ln (y))</td>
<td>(\ln \left(\frac{x}{y}\right) = \ln (x) - \ln (y))</td>
<td>(\ln \left(\frac{x}{e}\right) + \ln (x) = \ln (x^2))</td>
</tr>
<tr>
<td>(\ln (e^x) = x)</td>
<td>(\ln (1) = 0)</td>
<td>(e^{\ln (x)} = x)</td>
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</tbody>
</table>

\((x \pm a)^2 = x^2 \pm 2ax + a^2\)
Lines: slope of a line through two points \((x_1, y_1)\) and \((x_2, y_2)\) is \(a = \frac{y_2-y_1}{x_2-x_1}\)

The slope (=a)/intercept(=b) form: \(y = ax + b\)
The slope/point \((x_1, y_1)\) form: \(y - y_1 = a(x-x_1)\)
Two lines \(y = a_1x + b_1\) and \(y = a_2x + b_2\) are - parallel if \(a_1 = a_2\) and
- perpendicular if \(a_2 = \frac{-1}{a_1}\)

The distance between two points \(P_1(x_1, y_1)\) and \(P_2(x_2, y_2)\): 
\[|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\]

Parabolas \(y = ax^2 + bx + c\) \((a \neq 0)\) or \(y = a(x - d)^2 + e\) → extreme point \((d, e)\)
\(a > 0:\) parabola opens upward. \(a < 0:\) parabola opens downward.

\(y\)-intercept: \(y = c\)
\(x\)-intercept(s): solutions of \(ax^2 + bx + c = 0\). 
\(D = b^2 - 4ac > 0 \Rightarrow 2\) \(x\)-intercepts,
\(D = 0 \Rightarrow 1\) and \(D < 0 \Rightarrow 0\) \(x\)-intercepts.

Interval where \(y < 0:\) solve \(ax^2 + bx + c < 0\)

Functions — Properties of a function \(f(x)\) and its graph \(y = f(x)\)

1. \(D_f = \text{Domain } f:\) all permitted values of \(x\).
   \(\text{Range } f:\) all possible values of \(f(x)\) \((x \in D_f)\)

2. Vertical line test for a function \(f: x = a\) intersects the graph of \(f\) at most once

3. Even function: \(f(-x) = f(x).\) The graph can be reflected about the y-axis.
   Odd: \(f(-x) = -f(x).\) The graph of the function can be reflected about the origin.

4. Increasing function: \(x_2 > x_1 \Rightarrow f(x_2) > f(x_1)\)
   Decreasing: \(x_2 > x_1 \Rightarrow f(x_2) < f(x_1)\)

5. Composite function: \(f \circ g(x) = f(g(x))\) (“apply \(f\) after applying \(g\) to \(x\)”) 

6. Periodic function: \(f\) has a period \(c\) if \(c\) is the smallest number such that \(f(x+c) = f(x)\).

7. One-to-one function: \(x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)\) “a horizontal line intersects at most once”

8. The inverse \(f^{-1}\) of a one-to-one function \(f: f^{-1}(y) = x \Leftrightarrow f(x) = y\)
   Find the inverse by: 1. Checking if \(f\) is one-to-one  2. solving \(x\) from \(y = f(x)\) and 3. Interchanging \(x\) and \(y\).

Transformation of functions:

1. Shifting of a graph \((c > 0)\)
   - \(y = f(x) + c:\) shift \(y = f(x)\) \(c\) units upward
   - \(y = f(x) - c:\) shift \(c\) units downward
   - \(y = f(x - c):\) shift \(c\) units to the right
   - \(y = f(x + c):\) shift \(c\) units to the left

2. Stretching and reflecting of a graph \((c > 1)\)
   - \(y = cf(x):\) stretch the graph \(y = f(x)\) vertically by a factor \(c\)
   - \(y = \frac{1}{c}f(x):\) compress vertically by factor \(c\)
   - \(y = f(cx):\) compress horizontally by factor \(c\)
   - \(y = f(x/c):\) stretch horizontally by factor \(c\)
• \( y = -f(x) \): reflect about the x-axis
• \( y = f(-x) \): reflect about the y-axis

3. **Graphing the inverse function** \( f^{-1} \): Reflect the graph \( y = f(x) \) about the line \( y = x \).

**Functions - Important functions**

**Polynomial function** of degree \( n \in \mathbb{N} \):

\[
y = a_n x^n + \ldots + a_1 x + a_0 , \text{ if } a_n \neq 0
\]

\( n = 0 \) constant function: the graph is a horizontal line \( y = a_0 \)  e.g. \( y = -2 \rightarrow \)

\( n = 1 \) linear function: line \( y = a_1 x + a_0 \)
  \( \) e.g. \( y = -x + 1 \)

\( n = 2 \) quadratic function: the graph is a parabola \( y = a_2 x^2 + a_1 x + a_0 \)
  e.g. \( p(x) = -2x^2 + \frac{5}{2}x + \sqrt{3} \)

\( n = 3 \): cubic function, e.g.:

\[
p(x) = x^3 + (\sqrt{5} - 1)x - 1
\]

**Power functions**: \( f(x) = x^a, \ a \in \mathbb{R} \).

1. \( a \) is a positive integer
2. \( a \) is a negative integer
   e.g. \( a = -1 \) and \( a = -2 \):
   
   \[
   f(x) = x^{-1} = \frac{1}{x}
   \]

\[
\begin{align*}
   f(x) &= x^{-2} = \frac{1}{x^2} \\
   f(x) &= x^{\frac{1}{2}} = \sqrt{x} \\
   f(x) &= x^{\frac{2}{3}} = \sqrt[3]{x^2}
\end{align*}
\]
Rational functions: \( f(x) = \frac{P(x)}{Q(x)} \), for polynomials \( P(x) \) and \( Q(x) \).
Domain: \( Q(x) \neq 0 \)!
\[
f(x) = \frac{x^5 + 2x^3 - 6x + 1}{x^4 + 1}
\]

Algebraic functions: a function that is constructed from polynomials by algebraic operations: addition, subtraction, multiplication, division or taking roots. e.g:
\[
f(x) = \frac{\sqrt{x^2 + 1} - x + 3\sqrt{x^5 - 3x^2 + 4}}{x+4}
\]

Exponential functions \( f(x) = a^x \), base \( a > 0 \)
Domain = \( \mathbb{R} \), Range = \( (0, \infty) \) if \( a \neq 1 \)

Logarithmic functions \( f(x) = \log_a (x) \), for \( a = e \) \( f(x) = \ln (x) \)

Trigonometric Functions
f(x) = \sin(x) \quad \begin{aligned} f(x) &= \sin(x) \\ f(x) &= \cos(x) \\ f(x) &= \tan(x) \end{aligned}

<table>
<thead>
<tr>
<th>Function</th>
<th>Odd function</th>
<th>Even function</th>
<th>Odd function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>2π</td>
<td>2π</td>
<td>π</td>
</tr>
<tr>
<td>Range</td>
<td>[-1, 1]</td>
<td>[-1, 1]</td>
<td>(\mathbb{R})</td>
</tr>
</tbody>
</table>

Cosecant, secant and cotangent function

\[
f(x) = \csc x = \frac{1}{\sin x} \quad f(x) = \sec x = \frac{1}{\cos x} \quad f(x) = \cot x = \frac{1}{\tan x}
\]

Note: \(\sin^{-1} x \neq \frac{1}{\sin x}\)

(\(\sin^{-1} x\) is the inverse of \(\sin x\))

\(f(x) = \tan(x)\) has period \(\pi\):
\(f(x) = \tan(x)\) is one-to-one
if \(D_f = (-\frac{1}{2}\pi, \frac{1}{2}\pi)\):
Then the inverse function exists on \(D_f\):
\(f^{-1}(x) = \tan^{-1}(x)\)
$f(x) = \tan(x)$

The inverse function of $f(x) = \tan(x)$
Exercises Calculus Basic Math and Functions

Some exercises to improve your basic math skills (James Stewart, Appendices A, B and D)

A1 Rewrite without absolute value:  
   a. $\sqrt{5} - 5$  
   b. $|x + 1|$  
   c. $|x^2 + 1|$  

A2 Solve the inequality and give the solution set as an interval:
   a. $2x + 7 > 3$  
   b. $1 - x \leq 2$  
   c. $2x + 1 < 5x - 8$  
   d. $4x < 2x + 1 \leq 3x + 2$  
   e. $x^3 - x^2 \leq 0$  
   f. $\frac{1}{x} < 4$  

A3 Solve:  
   a. $|2x| = 3$  
   b. $|x + 5| \geq 2$  

A4 Solve for negative constants $a$, $b$ and $c$: $ax + b < c$  

A5 Solve, if possible, by factoring:  
   a. $x^2 - 10x + 25 = 0$  
   b. $x^2 = 8x + 9$  
   c. $9x^2 + 9x + 1 = 0$  
   d. $x^2 = 3x - 10$  

A6 If a ball is kicked upward from the top of a building 128 ft high with initial velocity 16 ft/s, then the height $h$ above the ground $t$ seconds later will be $h = 128 + 16t - 16t^2$. During what time interval will the ball be at least 32 ft above ground?  

B1 Find the distance between the points $(1, 1)$ and $(4, 5)$  

B2 Find the slope of the line through the points P(-3, 3) and Q(-1, -6)  

B3 Find the equation of the line if:  
   a. Through $(2, -3)$ and slope 6  
   b. the $x$-intercept is 1 and $y$-intercept is -3  
   c. Through (-1, -2), perpendicular to the line $2x + 5y = 8$  

D1 Find $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $\theta = a. \ 3\pi/4$  
   b. $9\pi/2$  
   c. $5\pi/6$  

D2 Find $\cos \theta$ and $\tan \theta$ if $\sin \theta = 3/5, 0 < \theta < \pi/2$  

D3 If in a right angle triangle the hypotenuse has length 10 and $\theta = 30^\circ$, compute the length of the adjacent side  

D4 Find all values of $x$ in $[0, 2\pi]$ satisfying:  
   a. $2\cos x - 1 = 0$  
   b. $2\sin^2 x = 1$  
   c. $\sin x < \frac{1}{2}$  
   d. $\sin(\frac{x}{2}) = \frac{\sqrt{3}}{2}$  

Test your basic math skills (compute the result yourself and choose the correct answer a, b, c or d)  

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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</thead>
<tbody>
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<td></td>
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</tbody>
</table>

Write $-\frac{1}{5} = \frac{a}{b}$ (where $a$ and $b$ are the smallest possible positive integers)  

Answer:

Exercises for Functions and their properties

1.1 Find the domain of the function
   a. \( f(x) = \frac{x^2 + 2}{x^2 - 1} \) b. \( f(t) = \sqrt[3]{t - 1} \)

1.2 Find the domain and the range and sketch the graph of the function
   a. \( g(x) = \sqrt{x - 5} \) b. \( f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases} \)

1.3 Determine whether \( f \) is even, odd or neither. If \( f \) is even or odd use symmetry to sketch its graph
   a. \( f(x) = x^{-2} \) b. \( f(x) = x^2 + x \) c. \( f(x) = x^3 - x \)

1.4 Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential or logarithmic function:
   a. \( f(x) = \sqrt[3]{x} \) b. \( g(x) = \sqrt{1 - x^2} \) c. \( h(x) = x^9 + x^4 \) d. \( k(x) = \frac{x^2 + 1}{x^3 + x} \)
   e. \( r(x) = \tan 2x \) f. \( s(x) = \log_{10} x \)

1.5 Suppose the graph of \( f \) is given (e.g. \( f(x) = x^2 \)). Write equations for the graph that are obtained from the graph \( y = f(x) \) as follows:
   a. Shift 3 units upward b. Shift 3 units downward c. Shift 3 units to the right d. Shift 3 units to the left. e. Reflect about the \( x \)-axis. f. Reflect about the \( y \)-axis g. Stretch vertically by a factor of 3 h. Shrink vertically by a factor of 3.

1.6 Given is the graph \( y = x^2 \). Sketch it and also
   a. \( y = f(x + 2) \) b. \( y = f(x) + 2 \) c. \( y = 2f(x) \) d. \( y = \frac{1}{2}f(x) + 3 \)

1.7 Graph each function by starting with the graph of a “standard function” and then applying an appropriate transformation:
   a. \( y = -1/x \) b. \( y = \cos (x/2) \) c. \( y = \frac{1}{x-3} \) d. \( y = 1 + 2x - x^2 \)

1.8 Find \( f + g \), \( f \cdot g \) and \( f/g \) if \( f(x) = x^2 + x \) and \( g(x) = 3x^2 - 1 \)

1.9 Find \( f \circ g \), \( g \circ f \), \( f \circ f \) and \( g \circ g \) and state its domain if:
   a. \( f(x) = 2x^2 - x \) and \( g(x) = 3x + 2 \)
   b. \( f(x) = \sqrt{x - 1} \) and \( g(x) = x^2 \)
   c. \( f(x) = \frac{1}{x} \) and \( g(x) = x^3 + x \)

1.10 Find \( f \circ g \circ h \) and state its domain if \( f(x) = x - 1 \), \( g(x) = \sqrt{x} \) and \( h(x) = x - 1 \)

1.11 Write an equation that defines the exponential function with base \( a > 0 \). State the domain and range if \( a \neq 1 \). Sketch the general shape of the exponential function for the cases:
   (1) \( a > 1 \) (2) \( a = 1 \) (3) \( 0 < a < 1 \)

1.12 Use the general shapes of 1.11 to give a rough sketch of:
   a. \( y = 2^x + 1 \) b. \( y = 3^{-x} \) c. \( y = 3 - 2^x \)

1.13 Find the exponential function \( f(x) = Ca^x \) whose graph is through the points \((1, 6) \) and \((3, 24) \)

Exercises inverse functions and logarithmic functions:

1.14 Find the exact value of each logarithm expression:
   a. \( \log_2 (64) \) b. \( \log_6 \left( \frac{1}{36} \right) \) c. \( 2 \log_2 (3) + \log_2 (5) \) d. \( e^{3 \ln(2)} \)

1.15 Express the given quantity as a single logarithm:
   a. \( 2 \ln(4) - \ln(2) \) b. \( \ln (x) + a \ln (y) - b \ln (z) \)

1.16 Solve each equation for \( x \):
   a. \( e^x = 16 \) b. \( \ln(x) = -1 \) c. \( 2^x = 5 \) d. \( \ln(x) - \ln(x-1) = 1 \)

1.17 a. What is a one-to-one function? b. How can you tell from the graph that the inverse function exists?

1.18 Determine whether \( f(x) \) is one-to-one if:
   a. \( f(x) = 7x - 3 \) b. \( g(x) = |x| \)

1.19 Find the formula of the inverse function:
   a. \( f(x) = \frac{1+3x}{5-2x} \) b. \( f(x) = \sqrt{2} + 5x \) c. \( f(x) = \ln(x + 3) \)

1.20 If a bacteria population starts with 100 bacteria and doubles every three hours then the number of bacteria after \( t \) hours is \( n = f(t) = 100 \times 2^{t/3} \)
   a. Find the inverse of this function and explain its meaning.
   b. When will the population reach 50000?
Solutions to the extra basic math exercises:

A

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<tbody>
<tr>
<td>1a.</td>
<td>5 - √5</td>
<td>1c.</td>
<td>x^2 + 1</td>
<td>2c.</td>
<td>x &gt; 3</td>
<td>2f.</td>
<td>x &lt; 0 or x &gt; 1/4</td>
<td>4.</td>
</tr>
<tr>
<td>1b.</td>
<td>x + 1 for x ≥ -1</td>
<td>2a.</td>
<td>x &gt; -2</td>
<td>2d.</td>
<td>-1 ≤ x ≤ 1/2</td>
<td>3a.</td>
<td>x = ±3/2</td>
<td>5a.</td>
</tr>
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<td></td>
<td>-x - 1 for x &lt; 1</td>
<td>2b.</td>
<td>x ≥ -1</td>
<td>2e.</td>
<td>x ≤ 1</td>
<td>3b.</td>
<td>x ≤ -7 or x ≥ -3</td>
<td>5b.</td>
</tr>
<tr>
<td>B</td>
<td>1, 5</td>
<td>2.</td>
<td>-9/2</td>
<td>3a.</td>
<td>y = 6x - 5</td>
<td>3b.</td>
<td>y = 3x - 3</td>
<td>3c.</td>
</tr>
<tr>
<td>D</td>
<td>1a.</td>
<td>sin(π/2) = 1/2, cos(π/4) = -√2/2, tan(π/2) = -1</td>
<td>2.</td>
<td>cos θ = 4/5,</td>
<td>tan θ = 1/2</td>
<td>4a.</td>
<td>1/√2, 1/2√3, 1/3√3</td>
<td>4c.</td>
</tr>
<tr>
<td></td>
<td>1b.</td>
<td>sin(9π/2) = 1, cos(9π/2) = 0, tan(9π/2): undefined</td>
<td>3a.</td>
<td>2, sin θ = 9/5,</td>
<td>tan θ = 1/3</td>
<td>4b.</td>
<td>1/2π, 1/2π,</td>
<td>5π/4, 3π/2</td>
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<td></td>
<td>1c.</td>
<td>½π, ½√3, ⅓√3</td>
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Answers to the test:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
<p>| | | | | | | | | | | | | | | | | |</p>
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<tr>
<td>b</td>
<td>c</td>
<td>a</td>
<td>d</td>
<td>b</td>
<td>c</td>
<td>a</td>
<td>d</td>
<td>c</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
<td>b</td>
<td>e</td>
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<td>19.</td>
<td>½</td>
<td>½</td>
<td>√3</td>
<td>⅓√3</td>
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Solutions for the Function-exercises:

1.1 a. D = { x | x ≠ ±1} b. D = (0,∞)

1.2 a. D = [5,∞), R = [0,∞) b. D = (-∞,∞), R = (-∞,∞)

1.3 a. Even (graph by reflecting about the y-axis) b. Neither c. Odd (graph by reflecting about the origin)

1.4 a. root b. algebraic c. polynomial, degree 9 d. rational e. trigonometric f. logarithmic

1.5 a. y = f(x) + 3 b. y = f(x) - 3 c. y = f(x - 3) d. y = f(x + 3) e. y = 1/f(x)

1.7 a. Start with y = 1/x and reflect it about the x-axis b. Start with y = cos x and stretch horizontally by a factor 2 c. Start with y = 1/x and shift it to the right 3 units.

1.8 f + g(x) = x^3 + 3x^2 + x - 1, f - g(x) = x^3 - 3x^2 + x + 1, f/g(x) = (x^3 + x)(3x^2 - 1) and f/g(x) = (x^3 + x) / (3x^2 - 1)

1.9 a. f o g(x) = 2(3x+2)^3 - (3x+2) + 2 , g o f(x) = 3(2x^2 - x)^2 + 2 , f o f(x) = 2(2x^2 - x)^2 - (2x^2 - x) and g o g(x) = 9x + 8 (D= (-∞,∞) for all)

1.10 f o g o h(x) = √x - 1 (D = [1,∞)) b. g o f(x) = x^2 (D = [1,∞))

1.11 f(x) = x^2 + 1, D = (-∞,∞), R = (0,∞) (the graph is increasing for a > 1, y = 1 for a = 1 and decreasing for 0 < a < 1)

1.12 a. Graph y = 2^x (it’s an increasing graph) and shift 1 unit upward b. y = 3 - x = (3/2)^x (decreasing graph)

1.13 a = 2, C = 3

1.14 a. 6 b. -2 c. 15 d. 8

1.15 a. ln(8) = 3 ln(2) b. ln(2y/3)

1.16 a. 4 ln(2) b. 1/e c. 5 + log_e(3) = 5 + ln(3)/ln(2)

1.17 a. If x1 ≠ x2 implies f(x1) ≠ f(x2) b. Every horizontal line intersects the graph at most once.

1.18 a. Yes b. No

1.19 a. f^(-1)(x) = 5x - 1 b. f^(-1)(x) = x^2 - 2 x ≥ 0 c. f^(-1)(x) = e^x - 3

1.20 a. f^(-1)(n) = 3log_2(n/100) b. After about 26.9 hours (f^(-1)(50000))